Mental Discipline Theory and Mathematics Education

GEORGE A. STANIC

There exists the commonsense notion that mathematics should be taught because the study of mathematics, in some general way, improves people's thinking "Correctness in thinking," said Robert Maynard Hutchins [1936], "may be more directly and impressively taught through mathematics than in any other way" [p. 84]. It is a notion with roots in the writings of Plato:

> Those who are by nature good at calculation are, as one might say, naturally sharp in every other study, and...those who are slow at it, if they are educated and exercised in this study, nevertheless improve and become sharper than they were. [*Republic*, Book VII, Grube translation, 1974, p 178]

The same dialogue from the *Republic* includes the suggestion that "calculation" is "not to be neglected" because one could not "easily find studies which give more trouble to the practising pupil" [p 178]. This idea of the benefits of learning to overcome difficult obstacles also persists today. In their *Case Studies in Science Education*, Stake and Easley [1978] summarized the views of teachers they interviewed as follows:

> Teachers said they could not change the fact that life is going to require youngsters to do a lot of things that do not make sense at the time, and that seem very difficult. Therefore, they argued that there should be a significant body of learning at every grade level which is difficult, which may not make sense at the time, but which has to be learned by every student. [16: 16]

Mathematics is often thought of as a part of the "significant body of learning" mentioned by those teachers.

The idea that the study of mathematics improves one's thinking and helps one learn to overcome difficult obstacles was part of the dominant curriculum theory of the 19th century, mental discipline theory. The role of mental discipline theory in the history of mathematics education is the focus of this article. More specifically, I will argue that we now remember mental discipline theory only as a caricature of what it was. It had its fanatic followers who believed it was best for children to study those subjects they most disliked, but it also had more sensible advocates who expressed a basic faith in human intelligence. The turn of the century is generally seen as the time when mental discipline theory was repudiated. It did begin to decline in influence at this time, but arguments about it, particularly in the field of mathematics education, continued well into the 20th century. And, although it will not be a focus of this article, present day manifestations seem to exist not just in the views of Stake and Easley's teachers but also in the views of a number of researchers who speak of the mind as if it did consist of the faculties identified by the mental disciplinarians

A description of the theory

Mental discipline theory represented a merger of the liberal arts tradition and faculty psychology. It was influenced by Cartesian and Aristotelian conceptions of the mind and was based upon a mind-body metaphor. Edward Brooks described the metaphor in the following passage from a work published in 1883:

> The mind is a spiritual activity and grows by its own inherent energies. Mental exercise is thus the law of mental development. As a muscle grows strong by use, so any faculty of the mind is developed by its proper use and exercise. An inactive mind, like an unused muscle, becomes weak and unskillful. Hang the arm in a sling and the muscle becomes flabby and loses its vigor and skill; let the mind remain inactive, and it acquires a mental flabbiness, that unfits it for any severe or prolonged activity. An idle mind loses its tone and strength, like an unused muscle; the mental powers go to rust through idleness and inaction. To develop the faculties of the mind and secure their highest activity and efficiency there must be a constant and judicious exercise of these faculties. [Brooks, 1883/1970, p 84]

According to Brooks, *faculties* are capacities "for a distinct form of mental activity" [p 78], and the three general classes of faculties which constitute the mind are the intellect, the sensibilities, and the will. Each of these general faculties consist of more specific faculties; for instance, Brooks claimed that the intellect was made up of the faculties of perception, memory, imagination, understanding, and intuition or reason. [pp. 78-79]

Some mental disciplinarians used slightly different labels for the faculties, but the discussion by Brooks is representative of mental discipline theory. The basic metaphor is to view the mind as if it were a muscle (or a collection of muscles). Just as a muscle requires exercise for development, so does the mind.

Mental discipline theory represented a merger between

the liberal arts tradition and 19th century faculty psychology because the metaphor just described does not require a particular subject or group of subjects with which to exercise the mind. Technically, from the viewpoint of faculty psychology, all that is required is sustained, extended, and rigorous study, with the content of study being relatively unimportant. This does not mean that just any content would do, but it does mean that the supporters of the liberal arts were successful in perpetuating the importance of the classical studies (e.g., Greek, Latin, mathematics) by formulating the rationale for these studies within the framework provided by faculty psychology.

For the mental disciplinarians, then, the mind was to be viewed if it were a muscle the "as if" quality of this metapho and any other curriculum metaphor must be stressed. Through metaphor, we are asked to view one thing (what is to be explained) as if it were something else (that with which we are familiar). These two components of the metaphor are not to be taken as literally the same thing. The metaphor provides a lens through which we may see what we want to explain in another way. Like a lens, the metaphor emphasizes certain points and diminishes (or even distorts) others. If we recognize this lens-like quality, metaphor can be a valuable asset to our thinking. It is crucial, however, to remember that metaphor, used in this way, not only helps but also shapes our perception; it "has imbedded in it an element of persuasion, and one who is not critically aware of the power of metaphor can easily become its victim" [Kliebard, 1982, p 15]

It is true that a number of mental disciplinarians did, in a sense, become "victims" of the mind-body metaphor by losing sight of its "as if" quality. Because of some of the extremes to which certain mental disciplinarians went, mental discipline is sometimes remembered only in caricature. We tend to remember most those mental disciplinarians who believed there was an inverse relationship between a subject's practical usefulness and its disciplinary value. For instance, arguing against Herbert Spencer, W H. Payne [1886] claimed that "it is not true that the practical value of a study is identical with its disciplinary value. On the contrary, it is much nearer the truth to assert that these two values are in an inverse ratio to each other" [p. 47]. However, as Bidwell and Clason [1970] indicate in the following comments on mathematics teaching based on "old faculty theory," mental discipline theory was implemented in a variety of ways:

> [Another] development...[in] mathematics teaching form Colburn's time to the late nineteenth century was extension of the use of manipulative objects in early number work. The emphasis on using physical things (cubes, beans, corn, buttons, etc., were suggested) was as great as, or greater than, this emphasis in any more recent widely used approach Thus, although we may consider old faculty theory archaic, at least in primary grades the application of the theory was not far from current practice. [p. 2]

In fact, based upon mental discipline theory, one could provide systematic answers for all five of the basic curriculum questions identified by Kliebard [1977, 1982]. These questions are:

1. What should we teach?
2. Why should we teach one thing rather than another?
3. Who should have access to what knowledge?
4. What rules should govern the teaching of what has been selected?
5. What should be the relationship among the various things we teach?

The mental disciplinarians' answers to the first two curriculum questions — What should we teach? and Why should we teach one thing rather than another? — reflect the merger of the liberal arts tradition with faculty psychology. Mathematics, Greek, and Latin were central in the curriculum of the mental disciplinarians because these subjects were considered ideal vehicles for developing the faculties of the mind. However, some mental disciplinarians such as Charles William Eliot, president of Harvard, argued that the study of certain other subjects, including modern languages and the natural sciences, could also provide mental discipline. Because the form of a subject was actually more important than its content, there were disagreements about what besides mathematics, Greek, and Latin should be part of the curriculum. In a sense, the merger of the liberal arts tradition with faculty psychology provided both a continuing justification for the liberal arts and — paradoxically — the basis for modifications of the curriculum as people began to argue for the value of other subject areas on the basis of their ability to strengthen the faculties of the mind as much as or more than the traditional liberal arts.

Since we must always teach some *thing* to some *one,* the third basic curriculum question is : Who should have access to what knowledge? This brings up the issue of curriculum differentiation. The mental disciplinarians' answer to this question of curriculum differentiation was based upon a fundamentally optimisitic view of human intelligence. Though they recognized the obvious differences that existed between people, what was more important was that all people were born with the same faculties, and it was the job of the school to develop those faculties. Because of this basic similarity, all students were to have access to the same knowledge. Like other mental disciplinarians, Eliot believed students should have equal access to knowledge in the curriculum, but he also argued that students be given some choice as to what they would study.

Since the way we approach the teaching of any knowledge inevitably affects what knowledge gets conveyed, the fourth basic questioon is: What rules should govern the teaching of what has been selected? The basis for the mental disciplinarians' answers to this and the last question — What should be the relationship among the various things we teach? — was, once again, the perceived similarity between people in terms of their basic faculties and how those faculties developed. That is, academic subjects were to be taught in such a way that rigorous, extended, and sustained study was maintained; and, especially according to Eliot and the Committee of Ten [NEA, 1983], all sub-

jects were to be taught in the same way to all people Sequence and balance in the curriculum were based on the natural unfolding of the faculties Therefore, "the subjects that were believed to strengthen the power of observation could be presented early in the school curriculum, and higher mathematics could be presented at the end when reasoning was supposed to emerge. Balance would be achieved by attending to all the faculties of the mind Since it was believed that faculties not developed at the proper time would atrophy or wither, it became the duty of curriculum makers to see to it that all the faculties of the mind were developed, thereby achieving intellectual harmony" [Kliebard, 1982, p. 12]

With answers like these to the five basic curriculum questions, mental discipline theory dominated educational thinking during the 19th century. Mathematics was viewed by the mental disciplinarians as having extraordinary potential for improving certain faculties of the mind, especially that of reason. According to Brooks [1883/1970], "The study, *par excellence,* for the culture of deductive reasoning is mathematics." [86]

The decline of mental discipline theory

Although mental discipline did begin to decline in influence at the turn of the century, experiments motivated by it and discussions aobut it continued for some time into the 20th century Experiments by James [1890], Thorndike and Woodworth [1901], and Thorndike [1924] played important roles in questioning mental discipline theory despite the fact that these experiments were problematic tests of the theory and other experimental evidence existed which was in conflict with their results.

In 1890, James reported an experiment involving the memorization of lines of poetry and concluded that training in memory did not help later memorization. Based on their transfer of training experiment conducted in 1901, Thorndike and Woodworth concluded that "it is misleading to speak of sense discrimination, attention, memory, observation, accuracy, quickness, etc." because "multitudinous separate individual functions are referred to by any one of these words. These functions may have little in common" [p. 249]. With his 1924 study, Thorndike struck a blow against the idea that particular subjects had the greatest disciplinary value. He argued that "the intellectual values of studies should be determined largely by the special information habits, interests, attitudes, and ideals which they demonstrably produce The expectation of any large differences in general improvement of the mind from one study rather than another seems doomed to disappointment.... Disciplinary values may be real and deserve weight in the curriculum, but the weights should be reasonable" [p. 98]

These experiments did not prove that mental discipline theory as a whole, or even in part, was false. In fact, it may be inappropriate to consider any curriculum theory as "false" in an absolute sense [Kliebard, 1982]. Because of the "as if" quality of metaphors, one could not claim, for instance, that the theory of mental discipline is false because the mind is not a muscle. The metaphor upon which a theory is based is a lens which someone can choose to look through or avoid When one accepts a particular metaphor, what is really being accepted consists of the assumptions which lie behind the metaphor and the consequences which result form its use. The curriculum theory which is based on metaphor is, essentially, an explanatory system. Either it reveals a lot and suggests many valuable questions to guide inquiry, or it suggests little and does not enhance our understanding. When one accepts a particular curriculum theory, what one is accepting is the perspective it provides for viewing the five curriculum questions. And no theory of curriculum is a purely scientific, objective, and value-free statement Regardless of what else is involved, no answer to the question of what we would teach can be given without an explicit or implicit value position being taken. Therefore, no curriculum theory can be said to have been proven "false" in some absolute sense.

The way people reacted to the experiments by James, Thorndike, and Woodworth was, however, quite important. Yet the very fact that Thorndike was still dealing with the issue in 1924 provides evidence that mental discipline theory did not suddently vanish at the turn of the century. Thorndike continued to discuss the disciplinary values of subjects even after his 1924 experiment. "Apparently," said Thorndike in 1925, "some careless thinkers have rushed from the belief in totally general training to the belief that training is totally specialized" [p. 365]. Clearly, however, he was not an advocate of mental discipline theory. "The advisable course," he said, "in estimating the disciplinary effect of any study, occupation or the like would seem to be to list as accurately as possible the particular situation-response connections made therein" [Thorndike, 1925, p. 421].

A factor in the decline of mental discipline theory which was, perhaps, even more important than the experiments was the great change taking place in American society. With the rapid growth of urbanization, industrialization, and immigration at the turn of the century came significant advances in school enrollment. Between 1890 and 1940, the population of the American high school increased twenty-fold. Many educators and citizens began to see the curriculum of the 19th century as inappropriate for the needs of the increasing, and increasingly diverse, school population. It was within such a climate that mental discipline theory declined in importance [Stanic, in press]

Mental discipline theory through the eyes of Charles William Eliot

A transitional figure in this era, was Charles William Eliot. Eliot espoused the basic tenets of mental discipline theory, but he also attempted to adapt somewhat to the changes taking place in society and in the school. In referring to discussions on the problems of the reorganization of secondary education beginning to take place near the end of the 19th century, Butts and Cremin [1953] claimed that "if any one beginning point of these discussions may be indicated, it was probably a speech of President Charles W Eliot [of Harvard] before the NEA's Department of Superintendence in 1888 entitled 'Can School Programs Be Shortened and Enriched?'" [p. 391]. Furthermore, they claimed that "Eliot's address was one of the more important stimuli to

the appointment of the three major committees of the National Educational Association during the 1890's" [p. 391] — those being the Committee of Ten [NEA, 1893], the Committee of Fifteen [NEA, 1895], and the Committee on College Entrance Requirements [NEA, 1899]. Though Eliot's place in this era should probably be placed within a wider context [see e.g., Krug, 1961, 1964; Sizer, 1964], he certainly was an important figure particularly in terms of the Committee of Ten's recommendations.

Still clearly a mental disciplinarian in his belief that developing the "power" of the mind is more important than the accumulation of a certain amount of knowledge, Eliot [1894/1909c] spoke of the "six essential constituents of all worthy education." "We must learn," he said, "to see straight and clear; to compare and infer; to make an accurate record; to remember; to express our thought with precision; and to hold fast lofty ideals" [p. 322]. Unlike some mental disciplinarians, Eliot did not believe that every student "must study the same things in the same proportions and to the same extent. On the contrary, representation of the different kinds of knowledge and mental action having been secured, the utmost possible provision should be made for the different tastes, capacities, and rates of progress of different children" [pp. 326-327]. Eliot's advocacy of the elective idea and expanding the number of subject areas considered appropriate for study began with a concern he had with the traditional college conception of a liberal education. At least as far back as 1884, Eliot argued that in addition to the accepted liberal studies of Latin, Greek, and mathematics, he would "add, as studies of equal rank, English, French, German, history, political economy, and natural science" [Eliot, 1884/1909d, p. 113].

Eliot's viewpoints were not controversial. Some of those views, as expressed in the report of the Committee of Ten [NEA, 1893], which is — ironically — often remembered as a conservative document, were harshly criticized by the "classicists":

> The most concentrated attack on the Report ... was to come at the end of 1894, and it came from the classicists. The American Philological Association denounced the Committee of Ten for recommending only two years of Greek in the classical program and created a Committee of Twelve to arouse the educational world to the defense of Greek. For three years this Committee of Twelve kept up its persistent attacks. It added to its fears for Greek its concern about the reduction by the Committee of the amount of Latin from five to four periods a week in the third and fourth years of high school. ... This attack from the classical side is of special interest in light of the later tendency to regard the Report as conservative or traditional and as fostering college domination over secondary schools. [Krug, 1961, pp. 14-15]

Eliot was somewhat of an unusual mental disciplinarian; nonetheless, through Eliot the basic ideas of the mental disciplinarians remained central in the discussions of educators at the turn of the century.

Perhaps the most powerful opponent of Eliot during this era was Granville Stanley Hall. Hall emphasized basing the curriculum on his own conception of the natural order of development in the child. He scoffed at Eliot's idea that particular subjects should be taught in the same way to all children, regardless of their destinations in life, referring to the idea as one of three "extraordinary fallacies" of the Committee of Ten [Hall, 1904]. Claiming "this principle" of the Committee of Ten does not apply to "the great army of incapables, shading down to those who should be in schools for dullards or subnormal children," nor to "geniuses," nor to "pupils that lie between these extremes" [p. 510], Hall argued that

> not only thus does this principle fail to recognize how vast as the mind itself the great departments of knowledge are, but how the pedagogic instinct almost inclines to the belief that there are perhaps almost as many ways of approach to them as there are minds, and that it would not be an insanely wild thesis for those not ignorant of the anthropology of youth to maintain that every individual mind has ideally its own best personal way of approach to every science, because each mind not only has, but is, its own method. [p. 511]

This idea that "each mind ... is its own method," when combined with Hall's concern for the different interests and probable destinations in life of particular individuals, essentially captures his view toward individualization of the curriculum based on the natural order of development of the child.

One must, however, be careful not to associate the idea of individualization exclusively with Hall and others like him. Eliot also advocated a form of individualization, arguing that

> it is a waste for society, and an outrage upon the individual, to make a boy spend the years when he is most teachable in a discipline the end of which he can never reach, when he might have spent them in a different discipline, which would have been rewarded by achievement. Herein lies the fundamental reason for options among school as well as college studies, all of which are liberal. A mental discipline which takes no account of differences of capacity and taste is not well directed. It follows that there must be variety in education instead of uniform prescription. [1884/1909, p. 119]

Eliot stressed "that children, like adults, are not alike, but infinitely different; that the object of education, as of life, is to bring out the innate powers and develop to the highest possible degree the natural and acquired capacities of each individual." [Eliot, 1892/1909b, p. 275]

The basic difference between Hall's and Eliot's conceptions of individualization lies in their different views of human intelligence and in their different emphases on students' probable destinations in life. While Hall [1904] was concerned with "the great army of incapables" in the schools, Eliot [1892/1909a] believed that "we Americans habitually underestimate the capacity of pupils at almost every stage of education, from the primary school through

university." [p. 260] While Hall [1904] spoke fondly of "European systems" where "choices between academic and other careers are made before the teens" [p. 510], Eliot [1905/1961] wondered whether "the American public intends to have its children sorted before their teens into clerks, watchmakers, lithographers, telegraph operators, masons, teamsters, farm laborers, and so forth, and treated differently in their schools according to these prophecies of their appropriate life careers." [p. 153] "Who," asked Eliot, "are to make these prophecies? Can parents? Can teachers? Can university presidents, or even professional students of childhood and adolescence? I have watched many hundreds of successful careers which no one — not even the most intelligent and affectionate parent — could have prophesied of the runners at twelve years of age; and I have always believed that the individual child in a democratic society has a right to do his own prophesying about his own career, guided by his own ambitions and his own capacities, and abating his aspirations only under the irresistible pressure of adverse circumstances." [p. 153] Because of his basic faith in human intelligence and his reluctance to assign probable destinations to particular pupils, Eliot argued that "uniformity should apply to the method of teaching and to the selection of the fundamental topics in each subject which is taught at all"; because of his belief in the unique talents of individual students, he argued that uniformity should not apply "to the selection of subjects by the individual pupil, or to the length of time that the individual pupil pursues each subject" [Eliot, 1905/1961, p. 152] Eliot's conception of individualization differed, then, in this fundamental way from that of Hall. And Eliot's work shows that, despite the decline in influence of mental discipline theory, the controversy over the theory only *began* at the turn of the century

The continuing controversy

Mathematics educators at the turn of the century were not blind to the changes taking place around them. Nonetheless, basic ideas associated with the mental disciplinarians were advocated by a number of leaders in the growing field of mathematics education. For example, Jacob William Albert Young [1906], who, along with David Eugene Smith, helped to define the field of mathematics education, claimed that "the facts of mathematics, important and valuable as they are, are not the strongest justification for the study of the subject by all pupils. Still more important than the subject matter of mathematics is the fact that it exemplifies most typically, clearly and simply certain modes of thought which are of the utmost importance to everyone." [p. 17] More specifically, Young referred to the importance of "understanding statements, noting facts, and making inferences." [p. 46] "As a first introduction to these arts," mathematics has a "unique value" because of "the certainty, simplicity, and possible gradation of its inferences." [p. 46] In addition "the skill gained from the study of mathematics...[is] available beyond the bounds of mathematics, whose form of reasoning is the ideal toward which all other reasoning strives; owing, however, to the simplicity and narrow range of mathematical inferences, mathematics can only give the beginning of the requisite practice." [p. 46] Of fundamental importance to Young, then, was the idea that "mathematics *properly* studied tends to strengthen, *does* strengthen, the power of thinking independently and accurately." [p. 40]

David Eugene Smith's belief in mental discipline theory remained strong throught his career. As he said in 1916:

> The attack upon mathematics on the ground that it has no general disciplinary value has thus far been abortive, scientifically. We have only to see how divergent are the results of various investigations to see the truth of this assertion. There has been not the slightest proof adduced that mental exercise requiring vigor of thought does not conduce to brain vigor, exactly as physical exercise conduces to bodily vigor. What has been shown is that certain specific claims, rarely made for many years past, seem not to be valid. [pp. 77-78]

Like Young and Smith, Isaac J. Schwatt never doubted the disciplinary benefits of the study of mathematics. Schwatt said without hesitation that "secondary mathematics has little application in the daily work of the average person"; however, he also said that "all men, whatever their vocation in life, are in need of those powers of mind which are awakened, strengthened and developed by proper training in mathematics" [1911, p. 102] "Unless a pupil is benefited by the mental training which the study of mathematical principles is capable of giving," said Schwatt, "the study of the subject must be considered a waste of time and energy." [1911, p. 103] Disagreeing with those who felt that the changing school population required significant changes in the curriculum, Schwatt believed that

> the methods of education ought to be the same whether the child is to become a laborer, a mechanic, or a professional man. The standard of morals and the ability to think right and to act right ought to be the same whatever may be the station or the occupation of a person in life.... Whatever advantages outside of the school the child of the well-to-do may enjoy over the the child of the poorer classes, each must have in school the same advantages for his physcial, moral and mental development, if he is to be able to make use in the same measure of the opportunities, which our country offers to all its citizens. [1908, pp. 55-56]

Schwatt was a classic mental disciplinarian who clearly viewed the form of a subject as more important than the content. He felt that "schools cannot teach all that we ought to know" but they should train students in "abilities and qualities of the mind" [1908, pp. 60-61]

Some mathematics educators, such as William Betz, went from having an open mind about criticisms of the disciplinary values of the study of mathematics to being more reluctant to accept such criticism. In 1908, Betz claimed that "the one-sided theory of mental discipline in the elementary [geometry] course must go. By 1913, he claimed that "contrary to the usual impression conveyed by superficial alarmists, it may be said confidently that "we have neither proved nor disproved the formal discipline

dogma" " [p 224]

The ideas of mathematics educators such as Young, Smith, Schwatt, and Betz were challenged by those who believed that mental discipline theory was wrong. In the American report of the International Commission on the Teaching of Mathematics, published in 1911 and 1912 by the United States Bureau of Education [International Commission, 1911a, 1911b, 1912], one of the committees claimed that "there is a disposition in school circles to deny that the importance of mathematical training is such as to warrant the place it has occupied and still holds." [International Commission, 1911a, p. 17] Furthermore, the committee stated that "there is, among school men, and in the public to-day, an unwillingness to accept the doctrine of formal discipline at it was formerly stated, so that it is no longer possible to justify the retention of any topic or subject by claiming for it merely great disciplinary value." [p, 22] David Snedden, perhaps the predominant critic of the role of mathematics in the school curriculum during this era, was a member of that committee.

The official "American Commissioners", Young, Smith, and W F. Osgood, disputed the claims of Snedden's committee in their final report to the International Commission. [International Commission, 1912] While stating that "mathematicians will accept and take to heart the results of scientific investigations that bear upon their science or its teaching", they warned against accepting too readily the conclusions of certain psychologists [p. 31] They argued that "with respect to the so-called "doctrine of formal discipline" in particular there is danger that results of psychologic research may be misunderstood and misapplied" and concluded that "few, if any, mathematicians who are conversant with the results of such experiments as have been made, and are sympathetic with their spirit, feel that aught has been as yet established which would require them to change their views of the value of the study of mathematics." [pp. 31-32]

Another challenge to the role of mental discipline theory in mathematics education came from Ernest C. Moore. [1917a, 1917b, 1917c] In two 1917 articles entitled "The Doctrine of General Discipline" and "Does the Study of Mathematics Train the Mind Specifically or Universally?" Moore used as evidence arguments by prominent educators from the past and psychological experiments to dispute "formal discipline" theory and to support his claim that "the only education we can possible believe in is specific education." [1917a, p. 322]

The second article, in which Moore [1917b, 1917c] focused more exclusively on mathematics, was to become particularly controversial. The article was a reprint of an address before the Association of Teachers of Mathematics in New England and was published in both the *Mathematics Teacher* [1917b] and *School and Society* [1917c]. In the article, Moore stated that his purpose was not to question the value of mathematics. As he said:

> The questions which we are to consider is not the question of the value of mathematics — nobody doubts its value to any one who has occasion to use it. The question we are to consider is whether it is to be regarded as unlike other studies which are valuable to those who use them and not of much account to those who do not, but is instead a preferred study which is to be pursued not for the sake of what we can do with it, but for the sake of what it will do to us. The value of mathematics as a tool, a human device for doing its part of the work of the world, is not disputed — it never has been. The value of mathematics as a universal discipline is not proven; it is disputed. [1917c, pp. 483-484]

Again citing historical authorities and using the results of certain "transfer of training" experiments (in one case actually using the results of an experiment to support his claim when the experimenter — Harold Rugg — had used the results as evidence *for* transfer of training), he concluded that mathematics does not train the mind "universally." "On the solid rock of specific education we can build and must build," said Moore, "for of the results of specific education we can be sure, but as for formal or general discipline, in the words of Professor Spearman: "The great assumption upon which education has rested for so many centuries is now at last rendered amenable to experimental corroboration — and it proves to be false"." [p. 491]

The fact that Moore saw such an address as appropriate and timely indicates that the issue of mental discipline, or formal discipline, or general discipline, or transfer of training, was still alive in 1917. Even though some people may want to draw fine distinctions between these terms, their overlapping usage in works of the era — with Moore's work being one good example — shows that most people used them synonymously. At the very least, they were used to refer to very similar concepts. This does not mean that everyone agreed on what the basic idea behind the labels actually meant in terms of classroom practice. In fact, some of the biggest disputes were over whether a particular experiment was an appropriate test of mental discipline. But, even in these disputes, the issue generally was not which label (e.g., mental discipline, formal discipline, general discipline, transfer of training) was appropriate; instead, given that the labels referred to basically the same idea, the argument was over what that *idea* would look like when operationalized. Certainly, some people attempted to make distinction by using different labels; but old ideas are sometimes given new labels in order to overcome negative connotations associated with the former labels.

Again, in itself, Ernest Moore's address shows a continuing controversy over mental discipline. But the further discussion which his address triggered provides even more evidence that mental discipline theory was still, in fact, controversial. Charles N. Moore [1917] took exception to the historical and experimental evidence given by Ernest Moore and provided contrary evidence in both areas. Charles Moore concluded that "the weight of evidence is decidedly with the advocates of disciplinary values." [p. 770] The argument continued in the pages of *School and Society,* with, in the order of their appearance, further discussions by E.C. Moore [1918b], C.N. Moore [1918], Robert E. Moritz [1918b], Paul J. Kruse [1918], Moritz

[1918a], and E C. Moore [1918a]. That it was not always a friendly discussion can be seen in Ernest Moore's opening line of response to Moritz: "Essential courtesy is not one of the general effects which his study of mathematics has imparted to your contributor, Professor Robert E. Moritz." [E.C. Moore, 1918a, p. 754]

Conclusion

The controversy over mental discipline theory was never really settled during the early 20th century, even though we now tend to act as though mental discipline theory was repudiated. Discussions about mental discipline, transfer of training, and the relationship of the teaching of mathematics to these ideas continued throughout the era. J.W.A. Young attempted to summarize the work done on "the transfer of training" and "the disciplinary value of studies." [Young, 1918a, 1918b, 1924] Writing in 1918, Young concluded that

> taken as a whole, the body of experiments tests discrete special abilities selected here and there from a very wide field and leaves many questions practically untouched, including the important one of training in reasoning. Almost none of the experiments have been repeated even a second time; the results are far from being scientifically established by numerous repetitions with concordant outcomes. What outcome there is gives distinct support to the belief that the results of training can be transferred, and that educational work has disciplinary as well as content value.

In a follow-up article, Young [1918b] claimed that "modern psychology itself is a monumental instance of transfer of mathematical ideals and thought processes." [p. 131] Furthermore, said Young:

> The edifice of experimental psychology has as one of its foundation stones the assumption that the same mental powers function in varying situations, that mental strength can be transferred from one activity to another and more or less different one. This is quite evident when we consider how psychology, following the example of mathematics, endeavors by making situations abstract, simple, and artificial, to control them more completely in the laboratory than can be done in every-day life.... If anyone finds the situations of mathematics artificial and unrelated to real life, how much more so are those of the psychological laboratory? Truly, the child has outdone its parent! The facts of mathematics have a high content value, but what is the content value of counting dots or memorizing nonsense syllables? Having no content value, what other value can the laboratory tests have than to give information as to how the mind would function in the complex situations of everyday, nonlaboratory life? The very existence of an organized body of such laboratory tests...voices the belief of its users in the applicability of the powers tested under circumstances that are superficially very different from those of the test. [p. 131]

In a 1924 edition of his classic, *The Teaching of Mathematics in the Elementary and Secondary School* (with a section on "Developments in 1913-1923), Young stated that his views had remained essentially unchanged over the years. He again used developments in the field of psychology to make his point. Referring to the army intelligence tests used during World War I, he said:

> No teacher of mathematics who believes that the study of his subject can contribute to the development of the pupil's general intelligence and who teaches with such development as one of his aims, need be disquieted by any assertions that psychology is against him. Even though he may see no whit of improvement in the pupil's mathematical thinking, or in his ability to handle the subject's more complicated processes, or in his appreciation of its wide range of usefulness, if the teacher succeeds only in giving the pupil such increased familiarity with the most elementary notions of the subject as will enable him with greater speed and accuracy to mark a cross in the square and not the triangle, and to draw a line from the second circle above the third below the fourth to the fifth, he may rest content with the knowledge that the best psychologists that the United States Government could summon to its aid will, in consequence of that improvement, rate that pupil higher in *general intelligence*. [p. 365]

Young's claims were again challenged [see — for a specific response to Young, 1918a, 1918b — Harvey, 1918]. However, even though he may not have captured the general mood of his era, Young and certain others concerned about mathematics education [e.g., Cajori, 1928; C.N. Moore, 1920; Moritz, 1918c; Painter, 1920] continued to represent a strong belief in the tenets of mental discipline theory. The editors of the *Mathematics Teacher* also refused to let the idea die and continued to publish articles such as Cassius Keyser's "The Human Worth of Rigorous Thinking" [1922] and Susanne Langer's "Algebra and the Development of Reason" [1931]. Certain writers attempted to adapt mental discipline theory to the changing times. J.H. Minnick [1921], then president of the National Council of Teachers of Mathematics, claimed it was fortunate that the best way to get the "disciplinary value" from mathematics study was to "get the other values in the most effective way." [p. 304] The mathematician N.J. Lennes [1923] attempted to disassociate mental discipline from faculty psychology, claiming that "it is far from safe to assume that the doctrine of transfer of the effects of learning, that is the doctrine of mental discipline, must stand or fall with the doctrine of the "faculties" " [p. 65] E.R. Hedrick [1928] argued for the transferability of mathematical "processes and ways of thinking." Charles Hubbard Judd [1929], the esteemed psychologist of the University of Chicago, advocated "a radical change in the long established methods of teaching" mathematics in order to emphasize the "informational aspects of the science" which he claimed serve as the basis for transfer. William Betz [1930] suggested that "the problem of transfer of

training is fundamentally one of good teaching." [p 197] Harold Fawcett [1935], Pedro Orata [1935, 1937], and Hedrick [1937] also spoke of the need to teach for transfer. The common thread that runs through all these discussions is the belief that the study of mathematics can improve one's thinking in some general way. And the point to be made is that although we look at mental discipline theory as archaic and remember it only in caricature, the theory has a lively history, particularly in the field of mathematics education.

References

Betz, W. [1908] The teaching of geometry in its relation to the present educational trend. *School Science and Mathematics, 8,* 625-633

Betz, W. [1913] What mathematical subjects should be introduced in the curriculum of the secondary school? *Mathematics Teacher, 5,* 218-233

Betz, W. [1930] The transfer of training, with particular reference to geometry. In W.D. Reeve (Ed.), *The teaching of geometry* (Fifth yearbook of the National Council of Teachers of Mathematics, pp. 149-198). New York: Columbia University, Teachers College, Bureau of Publications

Bidwell, J.K., & Clason, R.G. (Eds.) [1970] *Readings in the history of mathematics education.* Washington, DC: National Council of Teachers of Mathematics

Brooks, E. [1970] [Selections from *Mental science and methods of mental culture*] In J.K. Bidwell & R.G. Clason (Eds.), *Readings in the history of mathematics education* (pp. 76-90). Washington, DC: National Council of Teachers of Mathematics. (Original work published 1883)

Butts, R.F., & Cremin, L.A. [1953] *A history of education in American culture.* New York: Holt

Cajori, F. [1928] *Mathematics in liberal education.* Boston: Christopher

Eliot, C.W. [1909a] Shortening and enriching the grammar-school course. In C.W. Eliot, *Educational reform: Essays and addresses.* New York: Century. (Original work published 1892)

Eliot, C.W. [1909b] Undesirable and desirable uniformity in schools. In C.W. Eliot, *Educational reform: Essays and addresses.* New York: Century. (Original work published 1892)

Eliot, C.W. [1909c] The unity of educational reform. In C.W. Eliot, *Educational reform: Essays and addresses.* New York: Century. (Original work published 1894)

Eliot, C.W. [1909d] What is a liberal education? In C.W. Eliot, *Educational reform: Essays and addresses.* New York: Century. (Original work published 1884)

Eliot, C.W. [1961] The fundamental assumptions in the report of the Committee of Ten. In E.A. Krug (Ed.), *Charles W. Eliot and popular education.* New York: Columbia University, Teachers College, Bureau of Publications. (Original work published 1905)

Fawcett, H.P. [1935] Teaching for transfer. *Mathematics Teacher, 28,* 465-472

Grube, G.M.A. (Translator) [1974] *Plato's Republic.* Indianapolis: Hackette

Hall, G.S. [1904] *Adolescence: Volume II.* New York: Appleton

Harvey, N.A. [1918] The doctrine of formal discipline. *School Science and Mathematics, 18,* 536-538

Hedrick, E.R. [1928] The reality of mathematical processes. In J.R. Clark & W.D. Reeve (Eds.), *Selected topics in the teaching of mathematics* (Third yearbook of the National Council of Teachers of Mathematics, pp. 35-41). New York: Columbia University, Teachers College, Bureau of Pubications

Hedrick, E.R. [1937] Teaching for transfer of training in mathematics. *Mathematics Teacher, 30,* 51-55

Hutchins, R.M. [1936] *The higher learning in America.* New Haven: Yale University Press

International Commission on the Teaching of Mathematics. [1911a] *Mathematics in the elementary schools of the United States.* Washington, DC: Government Printing Office

International Commission on the Teaching of Mathematics. [1911b] *Mathematics in the public and private secondary schools of the United States.* Washington, DC: Government Printing Office

International Commission on the Teaching of Mathematics. [1912] *Report of the American commissioners of the International Commission on the Teaching of Mathematics.* Washington, DC: Government Printing Office

James, W. [1890] *Principles of psychology.* New York: Henry Holt

Judd, C.J. [1929] Informational mathematics versus computational mathematics. *Mathematics Teacher, 22,* 187-196

Keyser, C.J. [1922] The human worth of rigorous thinking. *Mathematics Teacher, 15,* 1-5

Kliebard, H.M. [1977] Curriculum theory: Give me a "for instance." *Curriculum Inquiry, 6,* 257-269

Kliebard, H.M. [1982] Curriculum theory as a metaphor. *Theory into Practice, 21,* 11-17

Krug, E.A. (Ed.) [1961] *Charles W. Eliot and popular education.* New York: Columbia University, Teachers College, Bureau of Publications

Krug, E.A. [1964] *The shaping of the American high school.* New York: Harper & Row

Kruse, P.J. [1918] A strategic retreat. *School and Society, 7,* 531-532

Langer, S.K. [1931] Algebra and the development of reason. *Mathematics Teacher, 24,* 285-297

Lennes, N.J. [1923] A mathematician on the present status of the formal discipline controversy. *School and Society, 17,* 63-71

Minnick, J.H. [1921] The aims of mathematics education. *Mathematics Teacher, 14,* 297-304

Moore, C.N. [1917] The inadequacy of arguments against disciplinary values. *School and Society, 6,* 767-770

Moore, C.N. [1918] The disciplinary value of the study of mathematics. *School and Society, 7,* 290-294

Moore, C.M. [1920] The claims of mathematics as a factor in education. *School Science and Mathematics, 20,* 438-442

Moore, E.C. [1917a] The doctrine of general discipline. *Education, 37,* 312-324

Moore, E.C. [1917b] Does the study of mathematics train the mind specifically or universally? *Mathematics Teacher, 10,* 1-18

Moore, E.C. [1917c] Does the study of mathematics train the mind specifically or universally? *School and Society, 6,* 481-491

Moore, E.C. [1918a] Does the study of mathematics train the mind specifically or universally? A reply to a reply. *School and Society, 7,* 754-764

Moore, E.C. [1918b] Mathematics and formal discipline — again. *School and Society, 7,* 137-140

Moritz, R.E. [1918a] The disciplinary value of mathematics. *School and Society, 7,* 739-740

Moritz, R.E. [1918b] Does the study of mathematics train the mind specifically and universally? A reply. *School and Society, 7,* 485-492

Moritz, R.E. [1918c] Mathematics as a test of mental efficiency. *School and Society, 7,* 59-60

National Educational Association. [1893] *Report of the Committee of Ten on Secondary School Studies.* Washington, DC: Government Printing Office

National Educational Association. [1895] *Report of the Committee of Fifteen on Elementary Education.* New York: American

National Educational Association. [1899] *Report of the Committee on College Entrance Requirements.* Chicago: Author

Orata, P.T. [1935] Transfer of training and educational pseudoscience. *Mathematics Teacher, 28,* 265-289

Orata, P.T. [1937] Transfer of training and reconstruction of experience. *Mathematics Teacher, 30,* 99-109

Painter, G.S. [1920] Mathematics as a study. *Educational Review, 59,* 19-40

Payne, W.J. [1886] *Contributions to the science of education.* New York: American

Schwatt, I.J. [1908] Modern tendencies in the teaching of mathematics. *Mathematics Teacher, 1,* 47-77

Schwatt, I.J. [1911] Is the average secondary school pupil able to acquire a thorough knowledge of all the mathematics ordinarily given in these schools? *Mathematics Teacher, 3,* 101-116

Sizer, T.R. [1964] *Secondary schools at the turn of the century.* New Haven: Yale University Press

Smith, D. E. [1916] What is to be the outcome? *Mathematics Teacher, 9,* 77-79

Stake, R. E., & Easley, J. A. [1978] *Case studies in science education. Volume II.* Washington, DC: Government Printing Office

Stanic, G. M. A. (in press) The growing crisis in mathematics educations during the early twentieth century. *Journal for Research in Mathematics Education*

Thorndike, E. L. [1924] Mental discipline in high school studies. *Journal of Educational Psychology, 15,* 1-22, 83-98

Thorndike, E. L. [1925] *Educational psychology: Volume II. The psychology of learning.* New York: Columbia University, Teachers College, Bureau of Publications

Thorndike, E. L., & Woodworth, R. S. [1901] The influence of improvement in one mental function upon the efficiency of other functions (I). *Psychological Review, 8,* 247-261

Young, J. W. A. [1906] *The teaching of mathematics in the elementary and the secondary school.* New York: Longmans, Green

Young, J. W. A. [1918a] Concerning experiments to test the transfer of training. *School Science and Mathematics, 18,* 1-10

Young, J. W. A. [1918b] Remarks on psychological investigations bearing on the disciplinary value of studies. *School Science and Mathematics, 18,* 130-138

Young, J. W. A. [1924] *The teaching of mathematics in the elementary and the secondary school.* New York: Longmans, Green

"Philosophy of history" often means what we can learn from the past to cope with the future. In our particular case it could mean what we can learn from the history of old mathematics for the sake of teaching young people. Strangely enough nobody has ever looked at the converse idea, that is, what can we learn from educating youth for understanding the past of mankind? ...

A century ago biologists were the first to assert the so-called biogenetic law: that ontogenesis is an abridged recapitulation of phylogenesis, that is, that the individual in its development briefly repeats the development of its kind. We know for sure that this law is not true in this trivial way. But neither is it true that the new generation starts just at the point where its predecessors finished. Our biological, social, and mental life starts somewhere in the past of our race, at stages where man was not yet what he is now. The young learner recapitulates the learning process of mankind, though, in a modified way. He repeats history not as it actually happened but as it would have happened if people in the past had known something like what we do know now. It is a revised and improved version of the historical learning process that young learners recapitulate.

"Ought to recapitulate," we should say. In fact we have not yet understood the past well enough to really give them this chance to recapitulate it.

Hans Freudenthal