From Should to Could: Reflections on Possibilities of Mathematics Teacher Education [1]

DAVID PIMM [2]

"Not seeking fulfillment, they are not swayed by desire for change." Lao-tzu

This article contains a discussion of some difficulties possibly inherent in mathematics teacher education, with examples taken from experience accumulated by members of the Centre for Mathematics Education (CME) at the Open University in Britain. Because of certain particular features of our circumstances and task, we may experience some of these tensions more acutely and be exposed to other issues that might not be so generally evident. Before exploring some of them, I first outline the situation in which we work.

The Open University is a national, tertiary, distance teaching institution for adults. For more than ten years, members of CME have been producing extensive distance education materials (comprising print, audio and video materials and software) for practising mathematics teachers at both elementary and secondary levels. Some of these are in the form of (200 hour) courses, while others involve a far less intensive time commitment and are considerably more specific in focus.

Our audience is practising classroom teachers of pupils 5-16, not initial training students.[3] We do not produce pupil materials for use in the classroom; we prepare materials for teachers to work on themselves (in both senses), as part of their increased preparation for being in the mathematics classroom. By distance education, I mean both that the materials' authors seldom, if ever, have direct contact with those teachers who take the courses each year, and also that the teachers work at the materials primarily on their own. There is usually some small amount of face-to-face tutoring and assignment marking built into our course provision, but this is localised geographically, and part of CME's efforts involve working with a tutoring force, attempting to convey our intents to and through them.

One consequence of these circumstances is to focus our attention on issues not only of presentation of ideas but also of ourselves; that is, examining our relationship to the teachers and what their experience of the authors' disembodied presence may be. Distance teacher education by all its means is not just face-to-face education *minus* something. It also has certain novel features, certain dynamics of its own.

Teachers: isolation and technology

Teachers often experience isolation, being cut-off from other teachers in their classroom (through this has its elements of safety too). It can be very difficult for them to get to see other teachers teaching. Looking at schools around the world can bring a sense of inevitability that it must necessarily be done by a sole adult with a group of children. But this isolation arises from certain classroom organisational practices, thus producing a solidified regularity of the sort that Gowin [1981, pp 26-8] terms "artifactual phenomena of interest". Even the mild flirtation with alternative practices in the sixties such as team-teaching, vertical groupings, and the like can serve to remind us that it *could* have been quite different and that it is quite an odd system (reminiscent of certain Western childrearing practices with the similar and often-commented-upon isolation of the parent).

Does it matter that teachers do not see other teachers at work? After all, most artisans do not watch other artisans. I think one major difference is that teachers do not produce products (current proposed teacher assessment practices notwithstanding), whereas you would see another artisan's products which would also allow for the possibility of a focused discussion about their common craft. My experience is that it is very hard to sustain a focused discussion on the theme of "how I teach".

Video technology can afford groups of teachers common access to the classroom of another. One consequence is the possibility of a common initial base for focused discussion. In our experience, too many discussions of teaching operate at a very general level, with maintainance of good social relations a high priority. While terms such as "discussion", "practical work" or "open-ended problem solving" may be regularly invoked, it is hard to get a strong sense of another's answer to the question "what do I do in my classroom?".

A second story that can be told is that, as with a small dramatic work, a video record of a teacher teaching allows the viewer to observe/identify with/reflect on some behaviour that is different from (but sufficiently like) that experienced by the viewer. Thus, it could allow a viewer to see-themself-through-seeing-someone-else. This could lead to a possibility emerging for the viewer that they had not previously perceived. ("I could do this" or "I don't always have to do that".)

Videotape of classrooms is currently the central, most powerful means members of CME have been employing during the last ten years. As with other didactic devices at both the school and tertiary levels, we need to be aware that such devices seldom do what we intend them to do — or certainly seldom *solely* that, frequently doing something else as well, unintended, and probably overlooked. We

have encountered deep and significant difficulties in working with this medium, which arise as teachers endeavour to communicate with one another about the tape (the dramatic parallel would be exchanging "interpretations" of or responses to the play). And I am concerned that sophisticated, more recent innovations, such as videodisc technology, may in fact accentuate rather than alleviate these difficulties.[4]

There are two common responses we have encountered from adults (both teachers and mathematics educators) as a result of their being shown a piece of classroom videotape; responses which can markedly interfere with our intents. I have labelled these responses *televisual* and *intimidated*.

The *televisual* response is:

> I am going to watch someone else teaching, which may well be enjoyable, but it is basically television. And *watching* is the primary activity. It may be educative but is passive. I always want to see more, to see what happens next, and I am applying sophisticated media criteria. I forget that I have seldom (if ever) seen television footage in real time (which is one reason why political demonstrations — which seem so long and tedious to participate in — come across as urgent and often violent on the news). My time sense is distorted. What am I supposed to do after the machine is switched off? [5]

This first difficulty has in part to do with expectations engendered by the use of a TV monitor. Television is a great particularising medium. Watching a few minutes of a recorded classroom can produce the illusion that you know the teacher well and can speak accurately concerning what the person is capable of. Yet videotape is always made in the present: seldom do you have any sense of the history of a class, of how they have been brought to where they are when we see them. Seeing a confident teacher at work, she may seem to be standing around, making a few casual remarks and dramatic things happen. I might be led to think that is all I need to do: try what I think of as "the same thing" and then I am surprised or distressed when I get markedly different results. What is amiss? The temptation is to say that *this* didn't work, while being very vague about what "this" is.

This *intimidated* response is:

> I am about to be shown Lawrence Olivier or Maggie Smith and (particularly in the current political climate) told or at best "invited" to emulate him/her. I feel threatened and I resist by immediately being hostile to something I don't like in the excerpt (and I can *always* find something). My focus then becomes one of finding fault (which is frequently contagious in a group setting), of separating myself from the person on the screen, frequently by making observations about things I would *not* have done or would have done differently. In particular, my focus is at the level of action, on what *was* done, and not on what I *might* do.

This second response has to do with expectations about the reasons why one is being shown the material and more importantly what one is expected to do with it. Some teachers starting our courses have expressed their concerns in the question: "What *should* I be doing in my classroom?" One of our intents is to have them leave the course asking "What *could* I be doing?". There is an analogy with the mathematical topic of probability. Probability does not exist in the material world. Everything that happens happens with 100% probability — so how can we develop a sense of possibility, a sense of what *might* or *could* have happened? Similarly, when showing videotape of teaching, only *actual* practice can be exemplified, never the possibilities, never what *might* have happened. So somehow, with videotape, the possibility for me has to arise from the actuality of seeing you.

Whenever trying to teach someone something, there is the difficulty of indicating what the teacher is attending to and what is seen as being available to be learnt. If you are learning French or the piano from me, should you be copying me and trying to imitate me in all my particularity? If not, how do you, the neophyte, decide where to place your attention or focus? A powerful anecdote comes from jazz saxophonist Sonny Rollins speaking about his mentor Charlie Parker:

> Bird was our avatar; we followed him around, and anything he did we wanted to emulate. ... We were very naive, of course, and didn't realise that Bird wanted us to pick up the music, but not the drugs. I had to learn my lesson some years later and it was quite a trip through Hades. [1992, p. 43]

The mythologist Joseph Campbell [1989, pp. 89-90] similarly writes:

> When anyone becomes a model for you, you tend automatically to imitate him. This is the spontaneous identification, and it's through such identification that something inside develops in you. The second phase is finding your own self. I think that wearing Oriental clothes or assuming Oriental names is not the correct way to go about it. You've displaced again; you have mistaken the clothing for the message. ... It's not in the manner of dress or speech; it's the manner of experience and illumination.

I don't want you to be me.[6] But there is a danger that that is what will be perceived as the teacher's task when on an in-service course, quite as much as a student's misperceived task in the classroom. And resisted, rightly. And I see that misperception as lying behind a lot of the resistance to videotape of classroom practice. Yet how can I as the teacher educator indicate what I think you might see or do? And as Campbell suggested, that identification at some level may be a crucial ingredient in order to see possibilities.

John Mason [1988, p. 33] has formulated Brousseau's notion of the didactic tension as follows:

> The *more* explicit I am about the behaviour I wish my pupils to display, the more likely it is that they will display that behaviour without recourse to the understanding which the behaviour is meant to indicate; that is, the more they will take the *form* for the substance.

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The less explicit I am about my aims and expectations about the behaviour I wish my pupils to display, the less likely they are to see the point, to encounter what was intended, or to realise what is was all about.

I think teacher educators frequently find themselves in the grip of that tension too.

But underlying both of these responses are questions about the nature and visibility of teaching/learning events, as well as the vocabulary used for describing salient moments (he *intimidated* her, she is *dominating* that group, those two aren't *thinking,* ..). We are so used to strongly interpretative language for describing classroom events that attempting to get back to a simpler, more observable set of descriptions may help.

One means by which we have tried to overcome or deflect these responses is by using techniques of reconstruction, while trying to distinguish between what John Mason has termed "giving an account of" what was seen and attempting to "account for" it. Most often disagreements occur over *accounts for*, where projective seeing is frequently imperceptible to the speaker. By trying to drive a wedge between the viewer and the screen, we hope both to educate the viewer about the nature of videotaped screen events as well as to provide a means for them to engage in focused and productive discussion about their own teaching experience and practice.[7]

There is also an assumption that a television camera is the passive recorder of what happened. Videotaping also produces the event (classroom or otherwise). and does not solely capture it. A videotape does not reproduce the original classroom event itself and no one — not even the teacher concerned — has privileged access to that. It is strongly arguable that *the* event never existed to be captured.

I find it helpful to think of watching videotape of teaching in terms of a Rorschach experience. I splatter my interpretations of teaching all over the screen, which after all is, at one level, just moving dots of light (*pace* Grice[8]) In so doing, they become externalised and more accessible to observation, both my own and those of others. I can find in my reactions things that I might never have owned up to in a cooler discussion. Consequently, whoever is working with a group on classroom videotape needs to find ways of deflecting attention away from the tape itself *after* it has served its initial purpose.

For instance, I showed a short piece of footage of a female secondary mathematics teacher restarting an investigation lesson by holding a report-back session both to establish where students were but also surreptitiously to generate a range of possible continuations for those who were completely stuck. The group of teachers I was working with were nearly all experienced, male, secondary heads of department and many of them commented on how she repeatedly turned her back on the class, and criticised her practice for doing so (rather than, say, enquiring how it might be that she didn't have to be concerned about turning her back on her class). I remarked that it didn't seem to be a problem for her, but was apparently a charged item for them, and we were able to move from a focus on the TV screen, to a focus on their practice and concerns

Action at a distance: distanced voices, isolated lives
The above observations and techniques arose and were developed in working with groups of teachers directly on videotaped material But in our long courses, we do are not in the position to do this.

Some of the original material (PME233, 1979) was staged in the studio and shot on film before being broadcast over the national television networks. The second course (EM235, 1982) offered many recorded illustrations of small groups of children engaged on tasks in quiet areas, working either with their teachers or someone from CME. During this period too there was a move away from broadcasting these materials as television *programmes* (with a consequent need for a certain self-contained structure and independence) to issuing them on videocassettes to be lent to teachers for the academic year.

The style of the subsequent course (ME234, 1989) changed and all of the almost four hours of videotape was taped live in actual classrooms (the remainder showing practising teachers working on mathematics at their own level, something that we have rarely seen elsewhere). This change arose in part as a result of teacher feedback comprising comments like: 'This is all very well, but where were the rest of the class of thirty and what were they doing while this intensely-focused work was being done?' With the permanent move to videotape distribution, we were also freed from the time and continuity constraints of television programming and decided to do without a voice-over or narrator directing the attention of the watcher. This mirrors changes in documentary technique away from the recently perceived-as-problematic, omniscient, never-seen voice directing attention or providing continuity or interpretative comments ("What you will see in this next episode is where ..")

ME234 also makes use of the videotape medium to talk about working with the medium For instance, at the outset, students are asked to watch a classroom sequence and then to view a taped discussion among a group of teachers attempting a focused reconstruction of the same sequence. This is followed by a different classroom sequence and a second, school-based discussion by the teacher's colleagues including the teacher we see himself This part lasts twenty-five minutes and is the teacher's introduction to the videotaped materials.

As a consequence, the more recent tapes are less coherent and free-standing in some sense, and indeed one is a video anthology of numerous 3-5 minute classroom excerpts. However, they are now more open to wider uses, and indeed we often ask teachers to look at the same sequence from a different perspective or as part of a different task With some innovations, these decisions have been maintained and enhanced in our new course (EM236, 1992). For instance, we have employed occasional voice-overs during segments, but only in the form of overlaying audio material from the teacher interviews (carried out before or after the actual class) about their particular classes, decisions or general ways of working

Classrooms: the individual and the group
Which classrooms do we show? We endeavour not to expose the teachers we record to the intimidated response,

partly by not promoting their lessons as "good practice", a phrase which entered the political vocabulary in the UK in the mid-eighties. Nonetheless, it would be disingenuous to say we just record practice. Many hours of tape are recorded and considerable effort and thought goes into the selection of the teachers recorded and the excerpt choices made. For instance, there has been concern about the loss of a sense of commonality in classrooms and a consequent loss or atrophying of whole-class ways of working [e.g. Tahta, 1991]. Consequently, *one* intent behind our choice of recording venue was to show particular teachers working with a variety of whole-class settings.

Why this particular concern? Partly it is to do with fashions and swings in mathematics education. But also, mathematics education is a discipline prone to the lure of single solutions, be it real problem solving, back-to-the-basics, Logo for all, or whatever. While understanding some of the historical and social forces behind individualisation, particularly in that most ostensibly individual of countries, the United States, it can be seen as much as a force towards isolation and separation than as an enabling allowing of difference. It seems to me at best a waste of potential and at worst near criminal to bring together groups of young people and then treat them as if the rest of the class were not there. The classic instrument of this hegemony of the individual is the single, moulded desk.

I recently spent some time in a US lab school watching a number of classes in various innovative programmes. What struck me most was how all of the class discussion was taking place based on mathematical work done outside of the classroom (usually the previous night), rather than discussing something that was being done in the here-and-now. The main mathematical activity was somehow off-stage, like the ghost of Hamlet's father: that is, referred to but almost never actually present.

One problem I think arises from seeing the lesson as the unit of teaching. Alan Bishop has talked about a related problem of "lessonising" the curriculum. Consequently, many prescriptions are about "the perfect lesson" (and the way Madeleine Hunter's (1979) work on teacher "effectiveness" has been used by certain North American school boards — for instance, to tie salary raises to teachers moulding themselves to the Hunter lesson structure — should be a salutary warning to us all) with the upshot that every lesson is to be the same! Safe, perhaps, but extremely dull. Once again, a layer of complexity has been confounded by an externally imposed layer of control.

Voice and text
So far I have focused exclusively on issues of using videotape. But a sizeable part of our materials are text-based. René Thom [1973, p. 196] has commented: "As soon as one uses a textbook, one establishes a didacticism, an academicism, even if the book be so written as to promote individual research". The question of text authors' relation to their audience is thoughtfully explored in John Fauvel's piece "Tone and the teacher: instruction and complicity in mathematics textbooks" [1991]. Issues of differential power and authority, as well as expectations concerning

books as sources of telling, compound difficulties of discussions of teaching.

Given the nature of our contact with our intended audience, this potentially dominating effect of text presents some particular challenges. One tension we continually face is between over-privileging our voice and having it present at all [9]. Examples of over-privileging might be informing the reader what they will see in the next video episode, or what they will make of an article they have been asked to read. Not having it present could arise from an abdication of informing them of what *we* value in certain sequences or why we chose to include what we did. This is a particular embodiment of the didactic tension I mentioned earlier. (One inherent problem is that any course only contains what it contains, and the audience has no conception of what decisions have been made along the way.)

For instance, much of the course material attempts to invoke teacher activity and reflection through tasks. In order to generate both continuity and depth, some response to our tasks must be available. We wish to "interact" as much as possible, despite the necessarily pre-determined nature of our print materials, as well as endeavouring to move away from a sole author's or a collective "course team's" voice [10].

One means we employ is to include comments and opinions of other teachers, by interviewing, for instance, the teachers whom we have videotaped for the course. Another is by offering a small group of teachers a trial run through the materials and incorporating their responses. Using interviews with mathematics educators and others on audiotape also helps to broaden the range of individuals to whom the teacher is exposed. Offering written comments including "One teacher said .. " allow us to simulate a discussion, as well as offering some responses not directly sanctioned as our own (and therefore perhaps less likely to be interpreted as "the right answer"). This introduction of a multiplicity of voices (and their concomitant multiple perspectives) is something which is essential for our aims with teachers who may well be studying in isolation.[11]

Lusting after change?
On occasion in this piece, I have referred to "our intents". One might be that teachers emerge from our courses more aware and reflective about how they teach and also more alert to other possibilities. One aim that we do not subscribe to is being desirous of their change. The word *change* causes me much concern at times [12] because it seems caught up in common attitudes to, and an apparent preoccupation with, change in teacher education. Mary Boole (a mathematics educator from back before there were such things) has written powerfully of *teacher-lusts*.

> The teacher (whether school-teacher, minister of religion, political leader or head of a family) has a desire to make those under him conform themselves to his ideals. Nations could not be built up, nor children preserved from ruin, if some such desire did not exist and exert itself in some degree. But it has its gamut of lusts, very similar to those run down by other facul-

ties. First, the teacher wants to regulate the actions, conduct and thought of other people in a way that does no obvious harm but is quite in excess both of normal rights and practical necessity. Next, he wants to proselytise, convince, control, to arrest the spontaneous action of other minds, to an extent which ultimately defeats its own ends by making the pupils too feeble and automatic to carry on his teaching into the future with any vigour. Lastly, he acquires a sheer automatic lust for telling people "to don't", for arresting spontaneous action in others in a way that destroys their power even to learn at the time what he is trying to teach them. What is wanted is that we should pull these three series tight so as to see their parallelism, and not go on fogging ourselves with any such foolish notion as that sex-passion is a lust of the flesh and teacher-lust a thing in itself pure and good, which may be legitimately indulged in to the uttermost

Few teachers now are so conceited as not to know that they have a great deal to learn, and that their methods need revising and improving, but the majority are seeking for improved methods of doing more of what they are already doing a great deal too much of. The improvement which they most need is to be brought under conviction, to be made to see their conduct, their aims, their whole attitude towards their pupils and their work, in the light reflected on them from those of the drunkard and the debauchee. [Boole; in Tahta, 1980, p. 11]

There are also teacher-educator-lusts. Are we any more aware of them than the teachers Boole is writing about? "...the majority are seeking for improved methods of doing more of what they are already doing a great deal too much of" — what a powerful claim! One teacher-education-lust I have both seen and experienced seems to be "thou shalt change". To go to conferences where teacher education is being discussed is to fall in with the change merchants.

I think we should examine equally critically our *need* (lust?) for the teachers we work with to change. [13] Their change is not our business; how, when and if they change is surely their concern alone. It is a continuation of the dangerous idiocy of assessing teachers (under the name of "accountability") through their students' results. If I as a teacher educator can only feel successful if the teachers I work with change (and in ways I want them to), I am setting up both myself and the teachers I am working with quite dramatically. I believe it is dangerous to lose sight of how difficult personal change can be — and we should not talk lightly or glibly about it, let alone expect or demand it

Acknowledgement

Notes

[1] A version of this article was presented at the 1992 AERA meeting, held in San Francisco.

[2] I am grateful to my colleagues in the Centre for Mathematics Education, many of whose ideas are discussed here.

[3] Though this will fundamentally change in 1994, when our first *pre-service* distance education materials will be offered.

[4] Videodisc technology involves computer control of branching and selection of a bank of images, whether photographic stills or continuous sequences perhaps with sound Such branching, for instance, ostensibly offers options and open possibilities at teacher decision points, and discs have been used in the context of both initial and in-service teacher education. Some discs I have seen also seem to involve presumptions of causality: if you had decided to do *this*, then *this* is what would have happened. One result can be to *reify* the actions themselves (e.g. taking a student to task on some action, dividing the class into pairs, ...) and render them independent of the individual. The moment that possibility enters, so does theatre, and any sense of a tape as a partial record of actual events disappears.

[5] See the paper by John Mason [1985] entitled 'What do you do when you turn off the machine?' It includes examples from working on mathematical film, computer animations and other software, as well as watching videotape of classrooms

[6] The film *The paper chase* [1971] deals very thoughtfully with this theme of a student's obsession with his teacher

[7] Barbara Jaworski [1989] has produced a pamphlet entitled *Using classroom videotape to develop your teaching* in an attempt to explore aspects of some of these problems

[8] Grice [1989, p. 37] offers his maxim of perspicuity — "avoid obscurity of expression' — and discusses the following example of a failure to be brief or succinct and its possible implications
> Compare the remarks: (a) Miss X sang "Home Sweet Home", and (b) Miss X produced a series of sounds that corresponded closely with the score of "Home Sweet Home".

[9] There is a similar current concern in school teaching of allowing the teacher (or mathematical tradition) a say at all, while not crushing student intuitions and sense of creativity by the weight of pre-existing mathematical thought Dick Tahta [1988, p. 311] writes:
> It is, of course, this element of *direction* that is the ideological crunch. For a variety of reasons, not always consistent, there is a developing disinclination by teachers, or perhaps by the influential teacher-training establishment, to be seen to be too heavily "directive'. Pupils are encouraged to work at "their own mathematics', at mathematics which is variously seen as being culturally dependent, gender independent, and struggling to be free from the shackles of tradition. [...] Although it is — rightly — sometimes considered a virtue to share with fellow learners, pupils are not always so obviously encouraged to share with tradition, with textbook, or with teachers

[10] Open University courses are written over a one- to two-year time span by a team of authors (sometimes as many as ten) One of our courses (PME233) used "I" as the text voice, whereas later ones have reverted to using the consensus-constructing "we". Producing texts results in the creation of a "voice" which can become the "OU's voice" because of the absence of the speaker

[11] This device is not restricted to mathematics education courses John Fauvel [1989] has written an article on sixteenth-century mathematics author Robert Record's text style, and compared it with that of Open University courses One of Record's rhetorical devices was to use dialogue form, introducing the voice of a student who "has a strong personality of his own" [Fauvel, p. 3]

[12] Another pair of words which causes me unease is 'model" and "modelling", words which seem almost ubiquitous in much current North American writing in teacher education. One difficulty is the coming together of *model* (in the sense of simplification, representation) and *model* (meaning optimal). There is also the image of modelling clothes (showing them — and you — off to best effect) Another is the shift from the noun "a model" to the

verb "to model". Blithely to say, "let me model that for you" on the one hand raises all the concerns I have written about in this article, and on the other can sound like a "politically correct" way of saying "let me show/tell you how to do it".

[13] Psychotherapists are generally more aware than teacher educators of the dangers of *wanting* the client to change (Recall the joke about how many therapists does it take to change a lightbulb. *Ans.* Only one, but the lightbulb has really got to want to change.) Claudine Blanchard-Laville [1992] has sensitively examined some of the psychic elements present in why certain mathematics teachers teach as they do and some of the difficulties they encounter. Clarke [1988] has written about similarities and differences between the teacher-pupil and therapist-client relationships. She offers the following maxim for consideration: "the other person can resist only if *you* are pushing". Tahta [1991] has written compellingly of the *need* of some teachers for their students to learn.

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As such, a Liar Paradox is an elaborate linguistic artifact. possible within the schema of language and in no way anomalous *as a linguistic object* Anomaly arises only when we try to relate it to the real world, say by trying to imagine a single proposition with content that would be at the same time true and false We feel intuitively that a sentence should have some kind of denotation, a feeling akin to the naive realist's belief that if there is a word *unicorn* then there ought to be unicorns.

But this is not so. Occurring on the secondary level of representation, sentences do not have to be congruent with reality, not even with the primary model of reality that they more directly represent All representational systems have properties of their own. some of which will not be found at all in whatever is supposedly represented. If, on the lexical level, this can lead to the creation of words whose referents do not or cannot exist, it can lead on the syntactical level to sentences about situations that do not or cannot exist

Derek Bickerton